Monte Carlo study of four body interaction Potts models

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We study three different four site interaction Potts models on the square lattice, with $q = 3$; 4; 5 states per spin, using the Wang-Landau entropic sampling algorithm. We use in our simulations lattices with linear sizes $L = 4$; 8; $12 \ldots$; 52 and periodic boundary conditions. With the aid of conventional finite size scaling methods and homogeneity arguments we give estimates on the infinite volume transition temperatures. The three models exhibit strong finite size effects. In addition, for $q = 3$, there are also indications for non-universal logarithmic-power correction to scaling terms associated with the specific heat. While the three and five states models show significant evidences of second and first order transitions, respectively, the $q = 4$ finite size behaviour is very ambiguous. This is particularly manifested in a double-peak reweighed energy pdf, in the vicinity of the transition point. Although one would expect such a scenario in first order transition models, the minima between the peaks does not decay exponentially with L. Moreover, the width of the distribution does not have a typical $1/\sqrt{N} = 1/L$ behaviour, which is clearly observed in the five states case. Thus, although we cannot exclude a first order transition, our analysis supports a continuous transition when $q = 4$. The critical correlation length and specific heat indices α and ν are then predicted for this marginal model. Finally, for the five states model, we estimate the correlation length. We do this independently in two ways: First, we perform a double Gaussian fit to the pdf for each lattice. We then measure the distance between the fitted two maxima (the positions of the energies of the ordered and disordered phases), and use a conventional reciprocal volume scaling relation. Second, we use a known formula which associates the volume dependent divergence of the specific heat to the latent heat. The results in both ways fairly agree.